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PLANE PROBLEMS OF THE STATIC LOADING OF A PIECEWISE HOMOGENEOUS LINEARLY ELASTIC MEDIUM*

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Integral equations are obtained for the plane problem of the static loading of a system of linearly elastic blocks interacting arbitrarily on contiguous boundaries. The derivation is based on an assumed special form of the Boundary Integral Equations (BIE) for one block. Problems on inclusions, systems of rigidly connected elastic blocks, blocks under mixed conditions on the outer boundary, and isolated, intersecting, tapering and growing cracks are included as special cases.

A number of problems of fracture theory, the mechanics of composite materials and mining geomechanics require the examination of problem of systems of elements (grains, composite components, mountain rock blocks and layers) that are contiguous and interact along sections of their boundaries. The component elements themselves (blocks) are often deformed elastically, while complex irreversible processes occur on the surfaces in contact. A similar situation occurs, say, during mutual movements of the grains, blocks, and layers, during crack formation on the boundaries of inclusions, during crack growth under compression conditions, etc. Direct application of existing BIE /1/ for elastic elements is far from optimal from the calculation point of view since the systems obtained turn out to be quite awkward and contain an excessive number of unknowns. Special forms of the BIE, enabling one to use the feature of contact problems that the forces on contiguous boundaries remain continuous and depend only on the difference in the displacements are more effective. This method, developed in the present paper, ensures that the final equations are compact almost halves the number of unknown quantities.

1. The plane problem of a system of p elastic elements (blocks or inclusions in an elastic matrix) bounded by contours L_j $(j = 1, \ldots, p)$ is considered. On the outer boundary we are given the loads or displacements, and we are given the relations governing the interaction on the contact sections. It is required to find the stresses and displacements in the elements and on their boundaries.

It is first necessary to agree on notation. The direction of traversal on the contours L_j is defined such that the j-th element is to the left. The outer domain relative to L_j is denoted by S_j^- and the inner one by S_j^+. Each of the closed contours L_j is the sum of sections L_{jm} on which the j-th element is contiguous to a total number s_j of elements m. In the case of inclusions separated by the matrix, $s_j = 0$. The direction of motion along L_{jm} is assumed to agree with the direction of traversal of L_j. Hence, L_{jm} and L_{mj} pass in opposite directions $(L_{jm} = -L_{mj})$.

The notation introduced is related to the number of the elements, and each of the contiguity sections is denoted twice (L_{jm} and L_{mj}). Later, a notation independent of the elements is also required for the set of contiguous sections and the outer boundaries. Hence, the outer boundary is denoted by N and it is considered that it passes so that the blocks remain on the left. This direction evidently agrees with the direction of motion over the sections of elements adjoining the outer boundary. The directions are fixed arbitrarily on the common boundaries of the blocks and the set of such segments is denoted by M. The sum $N + M$ corresponds to the common contour L.

The contour M separates left and right neighbourhoods at each of its points since the direction τ is fixed thereon. The limit values to the left of any quantity on M are marked with a plus, and on the right with a minus. The normal $\mathbf{n}$ is always considered to be directed to the right of the direction of motion.. Its direction is opposite at coincident points of the contours L_{jm} and L_{mj}, and agrees with the normal to L_{jm} of L_{mj} for an arbitrary point of the contour M on the boundary of the elements j and m depending on whether the motion along M at this point is in agreement with the direction of motion along L_{jm} or L_{mj}.

The pair of unit vectors $(\mathbf{n}, \tau)$ gives the direction of the axes of the local Cartesian coordinate system at each point of the contour L. The general Cartesian system xOy and the

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On the contact boundaries M the components $\sigma_{nn}, \sigma_{n\tau}$ of the force vector remain continuous on the axes of the local coordinate system, while the components of the displacements u_n^+, u_n^- and u_τ^+, u_τ^- experience the discontinuities $\Delta u_n = u_n^+ - u_n^-$, $\Delta u_\tau = u_\tau^+ - u_\tau^-$. The relation between $\sigma_{nn}, \sigma_{n\tau}$ and the discontinuity vector Δu_n, Δu_τ is assumed to be given at each point of M

$$\sigma_{nn} = A_{nn}(\Delta u_n, \Delta u_\tau), \quad \sigma_{n\tau} = A_{n\tau}(\Delta u_n, \Delta u_\tau) \tag{1.1}$$

or between the increments of these quantities

$$d\sigma_{nn} = B_{nn}\, d\Delta u_n + B_{n\tau}\, d\Delta u_\tau, \quad d\sigma_{n\tau} = B_{\tau n}\, d\Delta u_n + B_{\tau\tau} d\Delta u_\tau \tag{1.2}$$

The functions A_{nn}, $A_{n\tau}$ in Eqs.(1.1) can be non-linear, and the matrix with the coefficients $B_{nn}, B_{n\tau}, B_{\tau n}, B_{\tau\tau}$ can be non-negative definite (for the chosen axes and signs of the displacement discontinuities, energy absorption for mutual displacements at the contacts corresponds to negative definiteness). Moreover, the functions in (1.1) and the coefficients in (1.2) can be different at different points of M.

Relations (1.1) and (1.2) are fairly general in order to include the considerable number of contact conditions that may be realized in practice. The first corresponds to deformation, and the second to incremental variants in the rheology of rough interacting surfaces /2/. There are no differences between these cases for the formal part of the last exposition and it can be limited to a study of either of the variants. To be specific and to reduce the notation, we will start from (1.1) by keeping in mind that the change to incremental relationships is realized by adding the sign of the differential to the stress and displacement symbols. In addition, for the same reason the analysis is carried out as it applies to the problem of a system of blocks, although the majority of the results can also be extended to the problem of a matrix with inclusions. Therefore, the problem of finding the stress and displacement in a simply connected system of elastic blocks is formulated for given conditions on the outer boundary N and known interaction conditions (1.1) on the contacts M. The main difficulty is finding the contact values of the stress (or displacement) since, after they have been determined, the stress and displacement in each of the blocks is found by well-known methods of solving the fundamental problems for a simply-connected domain /3/.

2. We will first obtain and investigate the BIE of the necessary type for an arbitrary element j. When giving the principal vector f_j on its boundary, the Muskhelishvili functions φ_j and ψ_j, holomorphic in S_j^+ (within L_j) will satisfy the following trelationship on L_j:

$$\overline{\varphi}_j + \bar{t}\varphi_j' + \psi_j = \bar{f}_j, \quad t \in L_j \tag{2.1}$$

The necessary and sufficient conditions that $\varphi_j(t)$ and $\psi_j(t)$ be the limit values of the functions holomorphic in S_j^+, are given by the formulas

$$\varphi_j = \frac{1}{\pi i}\int_{L_j} \frac{\varphi_j\, d\tau}{\tau - t}, \quad \psi_j = \frac{1}{\pi i}\int_{L_j} \frac{\psi_j\, d\tau}{\tau - t} \tag{2.2}$$

After manipulation, by using the holomorphism of φ_j, ψ_j and φ_j' for their replacement by integrals of the form (2.2), we obtain the equation

$$l_j(t) + k_j\varphi_j - {}^1/_2 f_j - m_j(\bar{t}) = 0, \quad t \in L_j \tag{2.3}$$

$$l_j(t) = \frac{1}{\pi i}\int_{L_j} \frac{\varphi_j\, d\tau}{\tau - t}, \quad m_j(\bar{t}) = \frac{1}{2\pi i}\int_{L_j} \frac{f_j\, d\bar{\tau}}{\bar{\tau} - \bar{t}}$$

$$k_j\omega = -\frac{1}{2\pi i}\int_{L_j} \left(\omega\, d\ln\frac{\tau - t}{\bar{\tau} - \bar{t}} + \overline{\omega}\, d\,\frac{\tau - t}{\bar{\tau} - \bar{t}}\right)$$

For $f(t)$, in addition to the well-known formula /3/

$$f(t) = \int_{t_0}^{t} (-\sigma_{ny} + i\sigma_{nx})\, ds$$

we note the consequent dependence, suitable for the numerical solution of problems, under conditions of type (1.1)

$$f(t) = \int_{t_0}^{t} (\sigma_{nn} + i\sigma_{n\tau})\, dt \tag{2.4}$$

(t_0 is an arbitrary point in the domain considered, and s is the length of an arbitrary arc connecting the points t_0 and t).

The relationship (2.3) is transformed into Muskhelishvili equation /3/ if the first singular integral is replaced by φ_j using (2.2). However, such a substitution does not conserve equivalence: the eigenfunctions of (2.13) $\alpha i t, \beta + i\gamma$ (α, β, γ are real numbers) differ from the eigenfunctions of the Muskhelishvili equation studied in /4/. There are no other eigenfunctions besides those mentioned, as follows from the uniqueness of the stress field for loads given on the boundary.

The solution of the elasticity theory problem for the boundary condition (2.1) corresponds to any solution (2.3). Indeed, by introducing functions holomorphic outside of L_j

$$\Phi_j(z) = \frac{1}{2\pi i}\int_{L_j} \frac{\varphi_j\, d\tau}{\tau - z}, \quad \Psi(z) = \frac{1}{2\pi i}\int_{L_j} \frac{\bar{f}_j - \bar{\varphi}_j - \bar{\tau}\varphi_j'}{\tau - z}\, d\tau, \quad z \in S_j^- \tag{2.5}$$

(2.3) can be written in the form

$$[\Phi_j(z) + z\overline{\Phi_j'(z)} + \overline{\Psi_j(z)}]_e = 0 \tag{2.6}$$

where the subscript e denotes the limit value outside the contour L_j. According to (2.6), $\Phi_j(z)$ and $\Psi_j(z)$ solve the elasticity theory problem for domains exterior to L_j when there are no loads on its boundary L_j. It therefore follows that $\Phi_j(z) = \alpha_1 iz + C_1, \Psi_j(z) = C_2$, where α_1 is real and C_1, C_2 are complex constants. However, it follows from (2.5) that as z tends to infinity $\Phi_j(z)$ and $\Psi_j(z)$ tend to zero. Then $\alpha_1 = C_1 = C_2 = 0$ and the functions $\Phi_j(z)$ and $\Psi_j(z)$ equal zero for any z outside L_j. This means that $\varphi_j(t)$ and $\psi_j(t) = \bar{f}_j - \bar{\varphi}_j - \bar{\tau}\varphi_j'$ are limit values of functions, analogous to (2.5), that are holomorphic in S_j^+ and satisfy (2.1).

In passing, important properties of the solution are established for later. If it is assumed that the point t is outside the contour L_j (in particular, if it belongs to any other contour L_m), then

$$l_j(t) = 0, \quad t \in S_j^- \tag{2.7}$$

$$l_j(t) + k_j\varphi_j - m_j(\bar{t}) = 0, \quad t \in S_j^- \tag{2.8}$$

The last equation asserts the fact that $\Phi_j + z\bar{\Phi}_j' + \bar{\Psi}_j = 0$ outside L_j, which results from $\Phi_j(z)$ and $\Psi_j(z)$ being zero outside L_j. Differentiating (2.7) with respect to t, and taking the imaginary part, have in addition

$$\operatorname{Im} l_j'(t) = 0, \quad t \in S_j^- \tag{2.9}$$

Equation (2.8) means that (2.3) is satisfied not only at points of the contour L_j but also at all points exterior to it. The arbitrary function f_j outside L_j is here assumed to be equal to zero.

The solution (2.3) is determined to eigenfunction accuracy. In order to avoid them, it is sufficient to determine the values of $\varphi_j(z)$ and $\operatorname{Im}\varphi_j'(z)$ at an arbitrary internal point z_0. Assuming $\varphi_j(z_0) = \operatorname{Im}\varphi'_j(z_0) = 0$, we have

$$l_j(z_0) = 0; \quad \operatorname{Im} l_j'(z_0) = 0, \quad z_0 \in S_j^+ \tag{2.10}$$

Analogous equations hold for any exterior point by virtue of (2.7) and (2.9). Now we take into account that φ_j is expressed quite simply in terms of mechanical quantities /5/

$$\varphi_j = (2\mu_j u_j + f_j)/(\varkappa_j + 1); \quad u_j = u_{jx} + iu_{jy}$$

where μ_j is the shear modulus of the j-th block, $\varkappa_j = 3 - 4\nu_j$ for plane strain, and $\varkappa_j = (3 - \nu_j)/(1 + \nu_j)$ for the plane state of stress. Then (2.3), (2.7) and (2.9) take the form

$$\Lambda_j(t) + k_j\left(\frac{1}{2\mu_j} f_j + u_j\right) - \frac{\varkappa_j + 1}{4\mu_j}[f_j + 2m_j(\bar{t})] = 0, \quad t \in L_j \cup S_j^- \tag{2.11}$$

$$\Lambda_j(t) = 0, \quad \operatorname{Im}\Lambda_j'(t) = 0, \quad t \in S_j^- \tag{2.12}$$

$$\Lambda_j(z_0) = 0, \quad \operatorname{Im}\Lambda_j'(z_0) = 0, \quad z_0 \in S_j^+ \tag{2.13}$$

$$\left(\Lambda_j(t) = \frac{1}{\pi i}\int_{L_j}\left(\frac{1}{2\mu_j} f_j + u_j\right)\frac{d\tau}{\tau - t}; \quad f_j(t) = 0 \text{ when } t \in S_j^-\right)$$

Relation (2.11) is the desired BIE. It corresponds to the following requirements; only mechanical quantities occur in it and fictitious loads or displacements do not; it contains the displacement only under the integral sign; the BIE is satisfied identically for points outside the block if it is satisfied on the boundary. It is extremely desirable to obtain the BIE possessing such properties, even for the spatial problem of a block, since as is shown in the next section, the summation of such equations for all the blocks will yield a final relationship that contains only the force and the difference of the displacements.

3. A BIE of the form (2.11) can be written for each of the blocks ($j = 1, \ldots, p$). Taking into account that (2.11) is valid for any j even for points outside the j-th element, the relations can be added. Then, because of the continuity of the principal vector on the boundaries in contact, and taking account of the fact that the boundaries separating the elements pass twice in opposite directions ($L_{jm} = -L_{mj}$), an equation is obtained that is given on the total contour L

$$l(t) + k(a_1 f + \Delta u) - {}^1/_2 a_2 f - m(\bar t) = 0,\ t \in L \tag{3.1}$$

$$l(t) = \frac{1}{\pi i}\int_L \frac{a_1 f + \Delta u}{\tau - t}\, d\tau,\quad m(\bar t) = \frac{1}{2\pi i}\int_L \frac{a_3 f}{\bar\tau - \bar t}\, d\bar\tau \tag{3.2}$$

$$k\omega = -\frac{1}{2\pi i}\int_L \left(\omega\, d\ln\frac{\tau - t}{\bar\tau - \bar t} + \bar\omega\, d\,\frac{\tau - t}{\bar\tau - \bar t}\right),\quad \Delta u = u^+ - u^-$$

$$a_1 = \frac{1}{2\mu^+} - \frac{1}{2\mu^-},\quad a_2 = \frac{\varkappa^+ + 1}{2\mu^+} + \frac{\varkappa^- + 1}{2\mu^-},\quad a_3 = \frac{\varkappa^+ + 1}{2\mu^+} - \frac{\varkappa^- + 1}{2\mu^-}$$

where it is assumed that $u^- = 0,\ 1/\mu^- = 0$ on the outer contour. The direction of traversing L and the meaning of the plus and minus superscripts are mentioned in Sect.1.

The solution corresponding to the motion of the blocks as a rigid whole is eliminated by using additional conditions that are obtained by summing (2.12) taking (2.13) into account for fixed t, within one of the blocks and equal to z_0

$$l(z_0) = 0,\ \mathrm{Im}\, l'(z_0) = 0 \tag{3.3}$$

Substitution of the given values of f on the outer boundary and the values determined by (1.1) and (2.4) on the contact blocks into (3.1) and (3.3) results in an equation and conditions in which only the differences between the displacements Δu are known. This determines the advantage of (3.1) over other forms of integral equations of the plane problem for a system of interacting blocks.

4. Every solution of the elasticity-theory problem under consideration yields functions f and u that satisfy relationship (2.11) for each of the blocks. Addition of these equalities results in (3.1), i.e., every solution of the problem being studied about blocks satisfies the equation (3.1) obtained. The additional conditions (3.3) exclude the arbitrary displacement of the system as a rigid whole and extract the unique solution corresponding to a fixed state of stress for giving the stresses on the outer boundary.

The converse assertion that every solution (3.1) under the conditions (3.3) and the relation between f and Δu given by (1.1) and (2.4) is a solution of the problem for giving stresses on the outer boundary, is proved in a somewhat more complex manner.

Let Δu be a solution of (3.1). Then (1.1) and (2.4) determine f. It is first necessary to see that the principal moment of the forces applied to the boundary of any block, calculated by means of f, is zero. As is known /3/, this requirement is expressed by the equation

$$\mathrm{Re}\int_{L_j} \bar f\, dt = 0 \quad (j = 1, \ldots, p) \tag{4.1}$$

To prove (4.1) we write (3.1) in the form obtained by identity transformations

$$l + \bar t l' - a_2 \bar f = \frac{1}{\pi i}\int_L \frac{(a_1 \bar f + \Delta\bar u) + \bar\tau(a_1 f + \Delta u)' - a_3 \bar f}{\tau - t}\, d\tau \tag{4.2}$$

where the prime denotes the derivative of the function with respect to the argument.

Integrating (4.2) over the closed contour L_j, changing the order of the integrals on the right side, and noting that $a_2 + a_3 = (\varkappa_j + 1)/\mu_j$, according to (3.2), we obtain

$$\int_{L_j} [(\bar l - a_1 \bar f - \Delta\bar u)\, dt - (l - a_1 f - \Delta u)\, d\bar t] = \frac{\varkappa_j + 1}{\mu_j}\int_{L_j} \bar f\, dt \quad (j = 1, \ldots, p) \tag{4.3}$$

The left side of (4.3) is a purely imaginary quantity. Hence, the right side is also purely imaginary, i.e., Eqs.(4.1) to be proved are valid. It therefore follows that the function f found on solving (3.1) can be treated as the principal vector of the self-equilibrated loads acting on any of the contours.

A solution of the appropriate problems for the separate blocks exists, as is known. It determines the displacement of the blocks u_{*j} to the accuracy of their rigid motion. For each of them (2.11) holds. Summing (2.11) over all the blocks, we obtain an equation analogous to the initial equation for the same values of f, but containing the difference Δu_*, not associated with the loads by the dependences (1.1) in general. Comparing the difference between the initial and the obtained equations, we arrive at an equation of the form (3.1) for $f = 0$ in $\Delta u - \Delta u_*$. It is homogeneous, does not contain elastic constants, possesses a negative index $-c$ ($2c$ is the number of odd nodes of the contour L), and has a solution representing the difference between the rigid motions of adjacent blocks. Since there are no other solutions of the class h_{2c} (see /6/) for the homogeneous equation the displacements $\Delta u - \Delta u_*$ can differ from zero only by the difference between the rigid motions of the blocks. The displacements u_{*j} are themselves determined to rigid motion accuracy, i.e., Δu_* are determined to the accuracy of differences in the rigid motions. Hence u_{*j} can be selected so that $\Delta u - \Delta u_* = 0$. For such a selection $\Delta u = \Delta u_*$ and because Δu satisfies (1.1), Δu_* also satisfies this equation. In other words, f and u_{*j} yield the solution of the elasticity theory problem, which it was required to prove.

Solution (3.1) satisfying (3.3) determines the Muskhelishvili functions $\varphi_j(z)$ and $\psi_j(z)$ in an arbitrary j-th element

$$\varphi_j(z) = \frac{\mu_j}{\varkappa_j + 1} l(z), \quad z \in S_j^+ \tag{4.4}$$

$$\psi_j(z) = \frac{2\mu_j}{\varkappa_j + 1}\left[\frac{1}{2\pi i}\int_L \frac{(a_3 - a_1)\bar{f} - \Delta\bar{u}}{\tau - z}\, d\tau + \frac{1}{2\pi i}\int_L (a_1 f + \Delta u)\, d\frac{\bar{\tau} - \bar{z}}{\tau - z}\right] - \bar{z}\varphi_j'(z), \quad z \in S_j^+ \tag{4.5}$$

The stresses and displacements within the blocks are found by using $\varphi_j(z)$ and $\psi_j(z)$ by known formulas /3/.

5. Under the additional conditions (3.3), Eq.(3.1) is equivalent to the following equation without additional conditions, which is more convenient for computations

$$l(t) + k(a_1 f + \Delta u) - {}^1/_2 a_2 f - m(\bar{t}) - {}^1/_2 l(z_0) - (a^2/\bar{t})\,\mathrm{Im}\, l'(z_0) = 0, \quad t \in L \tag{5.1}$$

where a is an arbitrary real factor with the dimensions of length. It is introduced in order for the dimensions of the last term in (5.1) to agree with the dimensions of the other terms.

Every solution of (3.1) satisfying (3.3) obviously satisfies (5.1) also. Equivalence will be proved if the reverse is also established, that every solution (5.1) yields a solution of (3.1) under the additional conditions (3.3), and the solution of the initial problem.

The same computations that were utilized to derive (4.3) are performed for this proof.

It follows here that for any contour not enclosing the point z_0, the principal moment determined by the functions f satisfying (5.1) equals zero. Since the principal moment of the forces applied to N also equals zero, it follows that it equals zero even for a contour enclosing z_0. Then the same reasoning as was utilized in obtaining (4.3) results in the conclusion that the last term in (5.1) equals zero, i.e., the second of conditions (3.3) is satisfied. It then remains to consider equations analogous to (2.11) for each of the elements. They determine the functions u_j satisfying the first conditions of (2.12) and (2.13) in combination with f. Combining (2.11) and (2.12) and summing over j, we obtain an equation of the form (5.1), but for the differences Δu in the displacements which cannot satisfy (1.1). Subtracting it from (5.1) yields a homogeneous equation whose analysis is completely similar to that made above for (3.1) and results in the deduction that the solution (5.1) is a solution of the initial problem.

Relation (5.1) is applicable even when giving the displacements on part or on the whole of the exterior boundary N. On the sections where the displacements are known, the values of f here become unknown, while the last component is eliminated in (5.1). The penultimate component is eliminated if the equation is solved directly for the stress by using (2.4), and the value of f is determined at an arbitrary point of the outer contour.

In the case when the domain exterior to N is filled by an elastic material and the loads $\sigma_{x0}, \sigma_{y0}, \sigma_{xy0}$ act at infinity, (5.1) is also applicable with small changes. For $t \in N$ the component

$$[{}^1/_2(\sigma_{x0} + \sigma_{y0})t + {}^1/_2(\sigma_{y0} - \sigma_{x0} - 2i\sigma_{xy0})\bar{t}](\varkappa_0 + 1)/(2\mu_0)$$

is added on the left side of (5.1), and $\mu^- = \mu_0, \varkappa^- = \varkappa_0$ is considered for determining a_1, a_2, a_3 by means of (3.2), where μ_0, $\varkappa_0$ are constants corresponding to the outer infinite domain. Moreover, ${}^1/_4(\sigma_{x0} + \sigma_{y0})z$ is added in the first part of (4.4) for determining the functions $\varphi_j(z)$, $\psi_j(z)$ at points of the outer domain ($j = 0$), while the term ${}^1/_2(\sigma_{y0} - \sigma_{x0} + 2i\sigma_{xy0})z$ is added on the right side of (4.5), where $\varphi'(z)$ in (4.5) is calculated without the mentioned addition in (4.4).

6. Equation (5.1) (or (3.1) with the additional conditions (3.3)) includes a number of important special classes of contact and mixed problems.

Thus, when there are no discontinuities in the displacements ($\Delta u = 0$ on M) these relationships transform into an equation for f on M for rigidly connected blocks. For an infinite domain and just one block therein, an equation is obtained for an inclusion rigidly bonded to the matrix. For identical properties of the blocks that are in contact ($a_1 = a_3 = 0$ on M) the relationships become analogous to the equations in Δu, being distinguished by the fact that there is, however, a term $^1/_2 a_2 f$ expressible in terms of Δu on M outside the integral.

If two blocks in contact have identical elastic constants ($a_1 = a_3 = 0$) and there is no discontinuity in the displacements on certain sections of their common boundaries ($\Delta u = 0$), then the integrals over these sections vanish, and the equations can be considered for systems in which the blocks mentioned can be combined into one block along the part of the boundary on which the displacements remain continuous. Then the isolated sections of the discontinuities are isolated cracks. In general, the main vector on such sections turns out to be determined, apart from unknown constants which are found from the additional conditons for the solution to belong to the class H (see /6/).

If all the blocks have identical properties and discontinuities occur only along the isolated sections, then the equations correspond to a body with isolated slits. For an infinite domain, Eq.(3.1) is here transformed into the equation obtained in /7/ and used in /8,9/.

In general, any boundary M where the displacements experience a discontinuity ($\Delta u \neq 0$) can be considered as a crack. It is hence clear that the equations obtained refer not only to intersecting and isolated cracks but also describe their generation. They are indeed convenient for describing the successive stages of crack growth; hence additional sections are attached to the boundary M.

Different conditions can also be specified on the outer boundary N. Thus, Eq.(3.1) includes the case of the fundamental mixed problem and a problem in which the normal (tangential) component of the stress and the tangential (normal) component of the displacements are given. The equations enable even more complicated mixed problems to be solved when the conditions on N are described by relationships of the type (1.1).

The comparative simplicity and applicability of Eq.(5.1) for fairly braod classes of problems that are important in practice makes it worth compiling a program for solving it on a computer. A program that will assure a solution for 25 blocks interacting along the boundaries is quite realistic. If each is a square, and ten control points are selected on each side of the square, then the total number of real unknowns will not exceed 1300 after reduction to an algebraic system.

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